Intro to SUSY theory (d=4, N=1)

孙铮 @ 四川大学, pre-SUSY 2021, Aug. 2021.

references:

Books: John Terning, 《Modern Supersymmetry》 ~2006

Howard Baer & Xerxes Tata, 《Weak Scale Supersymmetry》 ~2006

Reviews: Matteo Bertolini, 《Lectures on Supersymmetry》 ~ 2017

https://people.sissa.it/~bertmat/teaching.htm

Stephen P. Martin, 《A Supersymmetry Primer》 ~2016

arXiv: hep-ph/9709356

More (old) references: Wess & Bagger, Bailin & Love, Dine, Freund, ...

Intro to SUSY theory $(d=4, N=1)$

Outline

* Intro: SM review, need for new physics.
* A simple SUSY model addressing problems of SM.
* Super-Poincare algebra and supermultiplets.
* Superspace, superfields and ~~the~~ SUSY Lagrangians.
* MSSM intro, ~~and~~ SUSY breaking, and mediation.

~~# A simple SUSY model~~

Conventions.

* metric $\eta^{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$
  Terning, Bertolini, Peskin ...... same
  Martin, Wess & Bagger ...... opposite.
* metric for spinors: (2x2 antisymmetric tensor)
  $\epsilon^{\alpha\beta} = \epsilon^{\dot\alpha\dot\beta} = -\epsilon_{\alpha\beta} = -\epsilon_{\dot\alpha\dot\beta} = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} = \epsilon$
* Pauli matrices $\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}$, $\sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$
  $\sigma^\mu{}_{\alpha\dot\alpha} = (\mathbb{1}, \sigma^i)$, $\bar\sigma^{\mu\dot\alpha\alpha} = (\mathbb{1}, -\sigma^i)$
  Gamma matrices $\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar\sigma^\mu & \end{pmatrix}$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & \\ & \mathbb{1} \end{pmatrix}$
* Poincare group $ISO(3,1)^+ \cong \mathbb{R}^4 \rtimes SO(3,1)^+$
  $\rightarrow$ Lorentz group $SO(3,1)^+$
  Spin group $Spin(3,1)^+ \cong \overline{SO(3,1)^+} \cong SL(2,\mathbb{C})$
  complexify $SL(2,\mathbb{C})_{\mathbb{C}} \cong SU(2)_{\mathbb{C}} \times SU(2)_{\mathbb{C}}$
* irreps (~~and~~ Casimir operator eigenvalues): (j_1, j_2) $\dim = (2j_1+1)(2j_2+1)$
  $(0,0)$: scalar ϕ (1)
  $(\frac{1}{2}, 0)$: left-handed spinor ψ_α (2)
  $(0, \frac{1}{2})$: right-handed spinor $\bar\chi^{\dot\alpha}$ (2)
  $\Big\}$ Weyl spinors, $(\psi_\alpha)^* = \bar\psi^{\dot\alpha}$
  $(\frac{1}{2}, \frac{1}{2})$: scalar + vector A^μ $(4 = 1 \oplus 3)$
  $(1,0)$, $(0,1)$ $\Big\}$ antisymmetric tensors (vectors) $F^{\mu\nu}$ or E^i, B^i $(3 \oplus 3)$

* Spinor bilinear contractions

$$\psi\chi = \psi^{\alpha}\chi_{\alpha} = -\psi_{\alpha}\chi^{\alpha} = \chi\psi$$

$$\bar{\psi}\bar{\chi} = \bar{\psi}_{\dot\alpha}\bar{\chi}^{\dot\alpha} = -\bar{\psi}^{\dot\alpha}\bar{\chi}_{\dot\alpha} = \bar{\chi}\psi$$

$$\psi\sigma^{\mu}\bar{\chi} = \psi^{\alpha}\sigma^{\mu}_{\alpha\dot\alpha}\bar{\chi}^{\dot\alpha} = -\bar{\chi}_{\dot\alpha}\bar{\sigma}^{\mu\dot\alpha\alpha}\psi_{\alpha} = -\bar{\chi}\bar{\sigma}^{\mu}\psi$$

* Dirac spinors $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$:

$$\Psi = \begin{pmatrix}\psi_{\alpha}\\ \bar{\chi}^{\dot\alpha}\end{pmatrix},\quad \bar{\Psi} = \Psi^{\dagger}\gamma^{0} = (\chi^{\alpha}, \bar{\psi}_{\dot\alpha}),\quad \partial\!\!\!/ = \gamma^{\mu}\partial_{\mu},\quad A\!\!\!/ = \gamma^{\mu}A_{\mu}$$

$$\mathcal{L}_{Dirac} = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi = i\bar{\psi}\bar{\sigma}^{\mu}\partial_{\mu}\psi + i\bar{\chi}\bar{\sigma}^{\mu}\partial_{\mu}\chi - m(\psi\chi + \bar{\psi}\bar{\chi})$$

Majorana spinors, $\bar{\chi}^{\dot\alpha} = \bar{\psi}^{\dot\alpha} = (\psi_{\alpha})^{*}$

$$\Psi_{M} = \begin{pmatrix}\psi_{\alpha}\\ \bar{\psi}^{\dot\alpha}\end{pmatrix},\quad \mathcal{L}_{Majorana} = \frac{1}{2}\bar{\Psi}_{M}(i\partial\!\!\!/ - m)\Psi = i\bar{\psi}\bar{\sigma}^{\mu}\partial_{\mu}\psi - \frac{1}{2}m(\psi\psi + \bar{\psi}\bar{\psi})$$

(Weyl spinors with Dirac and Majorana masses)

The Standard Model (SM)

symmetries → field contents → invariant action (Lagrangian) → phenomenology.
successfully verified by experiments.

* Symmetries: Poincare: $ISO(3,1)^{+}$ → Lorentz, $SO(3,1)^{+} \to Spin(3,1)^{+}$
internal, gauge: $SU(3)\times SU(2)\times U(1)$
global and discrete: flavor $U(3)$, C, P, T: Z_2^3

* Field contents

$s=1$, vectors: $A^{\mu} \to F^{\mu\nu}$, $A_{\mu} = A^{a}_{\mu}t^{a}$, $F_{\mu\nu} = F^{a}_{\mu\nu}t^{a}$, $F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + gf^{abc}A^{b}_{\mu}A^{c}_{\nu}$

gauge bosons → covariant derivative: $D_{\mu} = \partial_{\mu} - igA^{a}_{\mu}t^{a}$, $D\!\!\!/ = \gamma^{\mu}D_{\mu}$

$G^{\mu}: (8,1)_0$; $W^{\mu}: (1,3)_0$; $B^{\mu}: (1,1)_0$;

$s=1/2$, spinors: ψ

$$Q = \begin{pmatrix}u_L\\ d_L\end{pmatrix} = \begin{pmatrix}u_L & c_L & t_L\\ d_L & s_L & b_L\end{pmatrix}: (3,2)_{1/6};\quad U = u_R = (u_R, c_R, t_R): (3,1)_{2/3};$$

$$D = d_R = (d_R, s_R, b_R): (3,1)_{-1/3};$$

$$L = \begin{pmatrix}\nu_L\\ e_L\end{pmatrix} = \begin{pmatrix}\nu_{eL} & \nu_{\mu L} & \nu_{\tau L}\\ e_L & \mu_L & \tau_L\end{pmatrix}: (1,2)_{-1/2};\quad E = e_R = (e_R, \mu_R, \tau_R): (1,1)_{-1};$$

$s=0$, scalars: ϕ

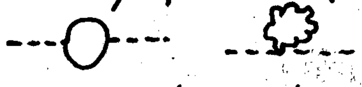
$$H = \begin{pmatrix}H^{+}\\ H^{0}\end{pmatrix}: (1,2)_{1/2}.$$

* Lagrangian: $\mathcal{L} = \sqrt{-g}\big(R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}D\!\!\!/\psi + (D_{\mu}\phi)^{\dagger}D^{\mu}\phi$
$\quad - \mu^{2}\phi^{\dagger}\phi - \lambda(\phi^{\dagger}\phi)^{2} - (y\bar{\psi}\phi\psi + h.c.)\big)$

(EWSB, Higgs mechanism, CSB, ...)

Need for new physics

* Hierarchy problem of Higgs mass.



$\Delta m_h^2 \sim g^2 \int \frac{d^4k}{k^2} \sim \Lambda_{UV}^2$; quadratic divergence.

* Gauge coupling unification in GUT.
* Cosmological constant. $\Lambda = \langle V \rangle \sim \Lambda_{UV}^4$
* Dark matter
* Strong CP problem
* Neutrino masses
* New physics : SUSY, extra dimension, axions, other new particles ...
* On theory side, Coleman - Mandula $\rightarrow$ Haag - Lopuszanski - Sohnius,
  symmetries of S-matrix : Poincare $ISO(3,1) \rightarrow$ superPoincare $\overline{Osp(4|N)}$
  $N=1$ or $N>1$ (extended SUSY)
* Ingredient of other theories.
  e.g. in string theory, SUSY + GSO project out tachyons.

A simple SUSY model.

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i \bar{\psi} \bar{\sigma}^\mu \partial_\mu \psi + F^* F .$$

* SUSY transformation, fermions $\leftrightarrow$ bosons.

$$\delta_\epsilon \phi = \epsilon \psi$$
$$\delta_\epsilon \psi_\alpha = -i (\sigma^\mu \bar{\epsilon})_\alpha \partial_\mu \phi + \epsilon_\alpha F$$
$$\delta_\epsilon F = -i \bar{\epsilon} \bar{\sigma}^\mu \partial_\mu \psi$$

  ϵ_α: infinitesimal spinor parameter.
  F: auxiliary field, no kinetics.
  introduced for closure of SUSY algebra.

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] X = (\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1}) X = i (\epsilon_1 \sigma^\mu \bar{\epsilon}_2 - \epsilon_2 \sigma^\mu \bar{\epsilon}_1) \partial_\mu X \ ; \quad X = \phi, \phi^*, \psi, \bar{\psi}, F, F^*$$

* $S = \int \mathcal{L} \sqrt{-g}\, d^4x$, invariant under SUSY : $\delta_\epsilon \mathcal{L} = \partial_\mu K_\epsilon^\mu$
  conserved Noether current : $\epsilon J^\mu + \bar{\epsilon} \bar{J}^\mu = \sum_X \delta_\epsilon X \frac{\partial \mathcal{L}}{\partial(\partial_\mu X)} - K_\epsilon^\mu$

$$\rightarrow J_\alpha^\mu = (\sigma^\nu \bar{\sigma}^\mu \psi)_\alpha \partial_\nu \phi^* , \quad \bar{J}_{\dot{\alpha}}^\mu = (\bar{\psi} \bar{\sigma}^\mu \sigma^\nu)_{\dot{\alpha}} \partial_\nu \phi$$

  supercharges $Q_\alpha = \sqrt{2} \int d^3x\, J_\alpha^0$, $\bar{Q}_{\dot{\alpha}} = \sqrt{2} \int d^3x\, \bar{J}_{\dot{\alpha}}^0$

* In QFT, canonical quantization of fields.

$$\rightarrow [\epsilon Q + \bar{\epsilon} \bar{Q}, X] = -i\sqrt{2}\, \delta_\epsilon X , \quad [P_\mu, X] = i \partial_\mu X$$

$\rightarrow$ superPoincare algebra $\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2 \sigma_{\alpha\dot{\alpha}}^\mu P_\mu$, $\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$

$$[Q_\alpha, P_\mu] = [Q_{\dot{\alpha}}, P_\mu] = 0$$

Including interactions, Wess-Zumino models.

~~(crossed out)~~

$$\mathcal{L} = \partial_\mu \phi^{*i} \partial^\mu \phi_i + i\bar{\psi}^i \bar{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i + \left(-\tfrac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i + h.c.\right)$$

* Superpotential $W = L^i \phi_i + \frac{1}{2} m^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k$, $W^i = \frac{\partial W}{\partial \phi_i}$, $W^{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$.

* EOM for F $\to F_i = -W_i^*$, $F^{*i} = -W^i$

$\to \mathcal{L} = \partial_\mu \phi^{*i} \partial^\mu \phi_i + i\bar{\psi}^i \bar{\sigma}^\mu \partial_\mu \psi_i - \frac{1}{2}(W^{ij} \psi_i \psi_j + h.c.) - W_i^* W^i$

* Scalar potential $V = W_i^* W^i$, minimize $V \to \langle V \rangle, \langle \phi_i \rangle, m_0^2, \ldots$ $W^{ij} \to m_{1/2}$

* DOF (real): on-shell (restricted by EOM): ϕ: 2, ψ: 2

off-shell: ϕ: 2, ψ: 4, F: 2

* e.g. $W = \frac{1}{3} g \phi^3$

$\to \mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\bar{\psi} \bar{\sigma}^\mu \partial_\mu \psi - g\phi\psi\psi - g^* \phi^* \bar{\psi}\bar{\psi} - g^* g (\phi^* \phi)^2$

$\to m_0 = m_{1/2} = 0$, $\langle \phi \rangle = \langle V \rangle = 0$. (cosmological constant suppressed by SUSY)

$\delta m_0^2 \sim$  $+$  (mass corrections canceled,

$\sim -2g^2 \int \frac{d^4k}{k^2} + 2g^2 \int \frac{d^4k}{k^2} = 0$ hierarchy problem solved by SUSY)

SUSY breaking, O'Raifeartaigh models, $\langle V \rangle > 0$ or $F_i \neq 0$

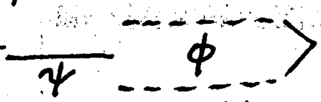
* e.g. $W = f\phi_1 + m\phi_2\phi_3 + \frac{1}{2} h \phi_1 \phi_3^2$, ($f, m, h > 0$ by phase rotations of ϕ_i)

$m^2 > fh \to$ vacuum at $\phi_1 \in \mathbb{C}$, $\phi_2 = \phi_3 = 0$, $V = f^2$

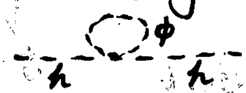
* Mass splitting: m_0^2: $0, 0, m^2, m^2, m^2 - fh, m^2 + fh$ (6 real)

$m_{1/2}$: $0, m, m$ (3 Weyl or 6 real)

supertrace $\mathrm{STr} M^2 = \sum_S (-1)^{2S+1} (2S+1) m_S^2 = 0$

m^2  $m^2 \pm fh$. (light fields after SUSY breaking)

* Hierarchy problem after SUSY breaking

$\delta m_h^2 \sim$  $+$ 

$\sim -2g^2 \int \frac{d^4k}{k^2 - m^2} + g^2 \left(\int \frac{d^4k}{k^2 - (m^2 + \delta m^2)} + \int \frac{d^4k}{k^2 - (m^2 - \delta m^2)} \right) \sim \frac{(\delta m^2)^2}{\Lambda_{UV}^2}$

※ more careful calculation: $\delta m_h^2 \sim m \log \Lambda$

logarithmic divergence OK for renormalization.

SUSY QED and Yang-Mills

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\bar\lambda^a\bar\sigma^\mu D_\mu\lambda^a + \frac{1}{2}D^aD^a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^a_\mu A^b_\nu,\quad D_\mu = \partial_\mu - igA^a_\mu t^a$$

* SUSY transformation: $\delta_\epsilon A^a_\mu = -\frac{1}{\sqrt{2}}(\epsilon\bar\sigma_\mu\lambda^a + \bar\lambda^a\bar\sigma_\mu\epsilon)$

D^a: auxiliary field. $\delta_\epsilon\lambda^a_\alpha = -\frac{i}{2\sqrt{2}}(\sigma^\mu\bar\sigma^\nu\epsilon)_\alpha F^a_{\mu\nu} + \frac{1}{\sqrt{2}}\epsilon_\alpha D^a$

$$\delta_\epsilon D^a = -\frac{i}{\sqrt{2}}(\epsilon\bar\sigma^\mu D_\mu\lambda^a - D_\mu\bar\lambda^a\bar\sigma^\mu\epsilon)$$

* For QED, Fayet-Iliopoulos term KD is also SUSY invariant.

Gauge-matter interactions.

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\bar\lambda^a\bar\sigma^\mu D_\mu\lambda^a + \frac{1}{2}D^aD^a\ (+KD)$$
$$+ D_\mu\phi^{*i}D^\mu\phi_i + i\bar\psi^i\bar\sigma^\mu D_\mu\psi_i + F^{*i}F_i + (-\tfrac{1}{2}W^{ij}\psi_i\psi_j + W^iF_i + h.c.)$$
$$-\sqrt{2}g\left((\phi^{*i}t^a\psi_i)\lambda^a + \bar\lambda^a(\bar\psi^i t^a\phi_i)\right) + g(\phi^{*i}t^a\phi_i)D^a$$

* EOM for auxiliary fields

$\rightarrow F_i = -W^*_i,\ F^{*i} = -W^i,\ D^a = -g\phi^{*i}t^a\phi_i\quad (D = -gq_i\phi^{*i}\phi_i - K \text{ for QED})$

* scalar potential $V = F^{*i}F_i + \frac{1}{2}D^aD^a \rightarrow$ F-term and D-term SUSY breaking

SuperPoincare algebra (N=1 SUSY)

$$\{Q_\alpha, \bar Q_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu,\quad \{Q_\alpha, Q_\beta\} = \{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = [Q_\alpha, P_\mu] = [\bar Q_{\dot\alpha}, P_\mu] = 0.$$
$$([Q_\alpha, M_{\mu\nu}],\ [\bar Q_{\dot\alpha}, M_{\mu\nu}],\ [P_\mu, P_\nu],\ [M_{\mu\nu}, P_\rho],\ [M_{\mu\nu}, M_{\rho\sigma}] = \cdots\cdots)$$

* Supermultiplets (irreps)

chiral: (ϕ, ψ, F); anti-chiral: $(\phi^*, \bar\psi, F^*)$; vector: $(A^\mu, \lambda; D)$

* DOF.

	ϕ	ψ	F	A^μ	λ	D
on-shell	2	2	0	2	2	0
off-shell	2	4	2	3	4	1

* SUSY invariant action (Lagrangian) built from supermultiplets.

Extended SUSY ($N>1$)

$$\{Q^I_\alpha, \bar Q^J_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu\delta^{IJ},\quad \{Q^I_\alpha, Q^J_\beta\} = 2\sqrt{2}\epsilon_{\alpha\beta}Z^{IJ},\quad \{\bar Q^I_{\dot\alpha}, \bar Q^J_{\dot\beta}\} = 2\sqrt{2}\epsilon_{\dot\alpha\dot\beta}Z^{IJ*}$$

central charge $Z^{IJ} = -Z^{JI}$, only possible for $N>1$

* Supermultiplets.

choose the frame $P_\mu = (E,0,0,E) \rightarrow \{Q^I_1, \bar Q^J_1\} = 4E\delta^{IJ}$

$|\Omega\rangle,\ \bar Q^{I_1}_1|\Omega\rangle,\ \bar Q^{I_1}_1\bar Q^{I_2}_1|\Omega\rangle,\ \cdots,\ \bar Q^{I_1}_1\cdots\bar Q^{I_N}_1|\Omega\rangle \sim \bar Q^1_1\cdots\bar Q^N_1|\Omega\rangle$

number of component states: 2^N

Superspace: a simplified notation to unify supermultiplets into superfields, with SUSY manifested.

* spacetime coordinates x^μ (mass dim $=-1$)
  Grassmann (spinor) coordinates θ_α, $\bar\theta^{\dot\alpha}$ (mass dim $=-\frac{1}{2}$)
* Berezin integral. $\int d\theta^\alpha = \int d\bar\theta_{\dot\alpha} = 0$, $\int d\theta^\alpha \theta_\alpha = \int d\bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha} = 1$
  $d^2\theta = d\theta^\alpha d\theta_\alpha$, $d^2\bar\theta = d\bar\theta_{\dot\alpha} d\bar\theta^{\dot\alpha}$, $d^4\theta = d^2\theta d^2\bar\theta$
  $\int d^2\theta\, \theta\theta = \int d^2\bar\theta\, \bar\theta\bar\theta = \int d^4\theta\, \theta\theta\bar\theta\bar\theta = 1$.
* superPoincare algebra represented as diffrential operators.
  $P_\mu = i\partial_\mu$, $Q_\alpha = i\partial_\alpha - \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu$, $\bar Q_{\dot\alpha} = -i\bar\partial_{\dot\alpha} + \theta^\alpha \sigma^\mu_{\alpha\dot\alpha}\partial_\mu$, $M_{\mu\nu} = \cdots\cdots$
  element of SuperPoincare group: $\exp(-i(\omega^{\mu\nu}M_{\mu\nu} + x^\mu P_\mu + \theta Q + \bar\theta\bar Q))$
* Covariant derivatives.
  $D_\alpha = \partial_\alpha - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu$, $\bar D_{\dot\alpha} = -\bar\partial_{\dot\alpha} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$
  $\{D_\alpha, Q_\beta\} = \{D_\alpha, \bar Q_{\dot\beta}\} = \{\bar D_{\dot\alpha}, Q_\beta\} = \{\bar D_{\dot\alpha}, \bar Q_{\dot\beta}\} = 0$

Superfields $S(x^\mu, \theta_\alpha, \bar\theta^{\dot\alpha})$

* The general expansion in θ_α, $\bar\theta^{\dot\alpha}$ (reducible rep. $S = \Phi + \bar\Phi + V$)
  $S = a(x) + \theta\psi(x) + \bar\theta\chi(x) + \theta\theta b(x) + \bar\theta\bar\theta c(x)$
  $\quad + \theta\sigma^\mu\bar\theta v_\mu(x) + \theta\theta\bar\theta\xi(x) + \bar\theta\bar\theta\theta\lambda(x) + \theta\theta\bar\theta\bar\theta d(x)$
* Chiral superfield: $\bar D_{\dot\alpha}\Phi = 0$
  $\Phi = \phi + \sqrt2\theta\psi + \theta\theta F + i\theta\sigma^\mu\bar\theta\partial_\mu\phi - \frac{i}{\sqrt2}\theta\theta\partial_\mu\psi\sigma^\mu\bar\theta - \frac14\theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\phi$
  $\longrightarrow$ chiral supermultiplet (ϕ, ψ, F)
* Anti-Chiral superfield: $D_\alpha\bar\Phi = 0$
  $\bar\Phi = \phi^* + \sqrt2\bar\theta\bar\psi + \bar\theta\bar\theta F^* - i\theta\sigma^\mu\bar\theta\partial_\mu\phi^* + \frac{i}{\sqrt2}\bar\theta\bar\theta\theta\sigma^\mu\partial_\mu\bar\psi - \frac14\theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\phi^*$
  $\longrightarrow$ anti-chiral supermultiplet $(\phi^*, \bar\psi, F^*)$
* Vector superfield: $V^* = V$, related to gauge groups.
  gauge transformation, Abelian: $V \to V + \frac{i}{2g}(\Omega - \Omega^*)$
  non-Abelian: $e^{2gV^a t^a} \to e^{-i\Omega^{a*}t^a} e^{2gV^a t^a} e^{i\Omega^a t^a}$
  choose the Wess-Zumino gauge to eliminate redundant DOF:
  $V = \theta\sigma^\mu\bar\theta A_\mu + i\theta\theta\bar\theta\bar\lambda - i\bar\theta\bar\theta\theta\lambda + \frac12\theta\theta\bar\theta\bar\theta D$
  $\longrightarrow$ vector multiplet (A_μ, λ, D)
  field strength. Abelian: $W_\alpha = -\frac14\bar D\bar D D_\alpha V$
  non-Abelian: $W_\alpha = W^a_\alpha t^a = -\frac{1}{8g}\bar D\bar D(e^{-2gV^a t^a} D_\alpha e^{2gV^a t^a})$
  $\qquad = -\frac14\bar D\bar D(D_\alpha V^a + ig f_{abc}(D_\alpha V^b)V^c)t^a$.

The SUSY Lagrangian

$$\mathcal{L} = \left(\frac{1}{4}\int d^2\theta \left(W_I^{a\alpha} W_{I\alpha}^a + W_J^\alpha W_{J\alpha}\right) + h.c.\right) + 2\int d^4\theta\, \xi_J V_J$$
$$+ \int d^4\theta\, \bar{\Phi}_i e^{2gV(i)} \Phi_i - \left(\int d^2\theta\, W(\Phi_i) + h.c.\right)$$
$$= \left(\frac{1}{4}\left[W_I^{a\alpha} W_{I\alpha}^a + W_J^\alpha W_{J\alpha}\right]_{\theta\theta} + h.c.\right) + \left[2\xi_J V_J\right]_{\theta\theta\bar\theta\bar\theta}$$
$$+ \left[\bar{\Phi}_i e^{2gV(i)} \Phi_i\right]_{\theta\theta\bar\theta\bar\theta} - \left(\left[W(\Phi_i)\right]_{\theta\theta} + h.c.\right)$$

* gauge group labels, I: non-Abelian, J: Abelian ($U(1)$)
* Gauge coupling. $2gV_{(i)} = 2g_I V_I^a t^a_{I(i)} + 2g_J q_{J(i)} V_J$
* All SUSY extension of SM fields are included in $\mathcal{L}$
* Expansion gives the same $\mathcal{L}$ in component fields as before.

MSSM

* Field contents

V: $G = (G^a, \tilde{G}^a)$: $(8,1)_0$; $W = \begin{pmatrix} W^\pm & \tilde{W}^\pm \\ W^0 & \tilde{W}^0 \end{pmatrix}$: $(1,3)_0$; $B = (B^0, \tilde{B}^0)$: $(1,1)_0$;

Φ: $Q = \begin{pmatrix} \tilde{u}_L & u_L \\ \tilde{d}_L & d_L \end{pmatrix}$: $(3,2)_{1/6}$; $\bar{U} = (\tilde{u}_R^*, u_R^\dagger)$: $(\bar{3},1)_{-2/3}$

$\bar{D} = (\tilde{d}_R^*, d_R^\dagger)$: $(\bar{3},1)_{1/3}$

$L = \begin{pmatrix} \tilde{\nu}_L & \nu_L \\ \tilde{e}_L & e_L \end{pmatrix}$: $(1,2)_{-1/2}$; $\bar{E} = (\tilde{e}_R^*, e_R^\dagger)$: $(1,1)_1$

$H_u = \begin{pmatrix} H_u^+ & \tilde{H}_u^+ \\ H_u^0 & \tilde{H}_u^0 \end{pmatrix}$: $(1,2)_{1/2}$; $H_d = \begin{pmatrix} H_d^0 & \tilde{H}_d^0 \\ H_d^- & \tilde{H}_d^- \end{pmatrix}$: $(1,2)_{-1/2}$

* MSSM superpotential

$$W = -y_u Q \epsilon H_u \bar{U} + y_d Q \epsilon H_d \bar{D} + y_e L \epsilon H_d \bar{E} + \mu H_u \epsilon H_d$$

$\longrightarrow$ Yukawa couplings in SM.

* Higgs potential comes from soft breaking terms.

SUSY breaking

* SUSY breaking at the vacuum: $Q_\alpha|0\rangle \neq 0$, $\bar{Q}_{\dot\alpha}|0\rangle \neq 0$
* $H = P^0 = \frac{1}{4}\bar{\sigma}^{0\dot\alpha\alpha}\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = \frac{1}{4}(Q_1\bar{Q}_{\dot 1} + \bar{Q}_{\dot 1}Q_1 + Q_2\bar{Q}_{\dot 2} + \bar{Q}_{\dot 2}Q_2)$

$\langle H\rangle = \langle 0|H|0\rangle = \frac{1}{4}(\|Q_1|0\rangle\|^2 + \|\bar{Q}_{\dot 1}|0\rangle\|^2 + \|Q_2|0\rangle\|^2 + \|\bar{Q}_{\dot 2}|0\rangle\|^2) \geq 0$

* Neglecting spacetime dependence and fermion condensations, $\langle H\rangle = \langle V\rangle$.
* $V = F^{*i}F_i + \frac{1}{2}D_I^a D_I^a + \frac{1}{2}D_J D_J \geq 0$, SUSY $\leftrightarrow V=0 \leftrightarrow F=D=0$

SUSY breaking $\leftrightarrow V>0 \leftrightarrow F\neq 0$ or $D\neq 0$. F-term and D-term.

SUSY breaking in a hidden sector.

* SUSY addressing problems of SM → MSSM or NMSSM, ...
* No sparticles is observed now. They are heavy or weakly coupled, different than SM particles. So SUSY is spontaneously broken.
* SUSY breaking by MSSM particles has problem of light fields.

m^2 SM MSSM, $m^2 \pm \delta m^2$ or SM MSSM

* SUSY must be broken in a hidden sector, and mediated to the MSSM sector.
* SUSY breaking: F-term (Wess-Zumino, O'Raifeartaigh)
  D-term (including Fayet-Iliopoulos term)

~~*Mediation~~

Mediation of SUSY breaking: gauge, gravity, anomaly.

* SUSY breaking $F_X \longleftrightarrow$ messengers $\longleftrightarrow$ MSSM
* Gauge mediation: messengers are chiral supermultiplets which are charged or non-trivialy rep. in SM gauge groups.

$$m_{soft} \sim \frac{\langle F \rangle}{M_{messenger}}$$

* Gravity mediation: invokes supergravity.

$$V = e^{\frac{K}{M_{Pl}^2}} \left(K^{\bar{i}j} F_i^* F_j - \frac{3}{M_{Pl}^2} W^* W \right), \quad -F^{*i} = D_i W = \partial_i W + \frac{1}{M_{Pl}^2} W \partial_i K$$

$\langle F \rangle$ → gravitino mass → soft masses $m_{soft} \sim \frac{\langle F \rangle}{M_{Planck}}$

* Anomaly mediation: through conformal anomaly.

$$m_{soft} \sim \frac{\langle F_\Sigma \rangle}{M}, \quad \langle \Sigma \rangle = M + \theta\theta F_\Sigma.$$